MULTIPLE RESOURCE LEVELLING IN NETWORKS

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By
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CERTIFICATE

This is to certify that the thesis entitled "Multiple - Resource Levelling in Networks" by Madan Pant has been carried out under my supervision and has not been submitted elsewhere for the award of a degree.

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SYNOPSIS

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"MULTIPLE RESOURCE LEVELLING IN NETWORKS"

(Levelling of resources in the network of a project is essential to attain a relatively stable resource demand along the length of the project. A stable resource requirement means low schedule costs and good public relations. In case of manpower levelled resource requirements usually boost the employee morale.)

In this work, a heuristic resource levelling model is proposed for multiple resource networks. (This model consists of a CPM routine followed by three heuristic routines, viz., Forward Search routine, Back-tracking routine, and Crashing of non-critical activities routine. It also has the provision of generating several alternate schedules among which the one giving the best levels is accepted as the final schedule) Three levelling policies have been suggested for the different types of project conditions occuring in practice. In the first policy, the costs contributed by the peak requirements of different resources are minimized. The second policy attempts to minimize the costs arising due to frequent hiring and firing of resources. The last policy incorporates the features of the first two policies.

The proposed levelling model and policies were applied to a number of multi-resource problems. (The result showed that this model gave good resource levelling for most of these networks.)

CHAPTER 1

INTRODUCTION

1.1 The Resource Scheduling Problem

In this age of shortages, when resources are becoming scarce all over, the need for an efficient utilization of resources is felt in all projects. This is achieved by better planning, coordination and control of various activities which constitute a project. The Operations Research techniques such as Linear Programming, Dynamic Programming, and Non-Linear Programming are found cumbersome, difficult to handle, and inefficient for these problems. A comparatively recent development, the network planning technique, has found wide acceptance in the field of resource management.

The application of network analysis to problems of planning and control started in 1957, apparently independently by two different groups. One, at the Du Pont Company, started a project designed to control the maintenance of chemical plants. The outgrowth of this project was the CPM (Critical Path Method). Independent of the Du Pont effort, the U.S. Navy undertook in 1957 the task of planning and controlling the Polaris project. The result was the PERT (Program Evaluation and Review Technique) which was used to plan and control the Polaris project with outstanding success.

In their original forms, both PERT and CPM were essentially time-oriented. As scheduling tools, they enabled managers to minimize project duration, assuming that the various resources required for the project completion were available. In practice, however, project completion requires the use of various resources, which are often limited in availability. These may directly influence planning objectives, time estimates, scheduling and progress control. When activities require resources for their execution (e.g., manpower, materials, equipment, capital etc.) that are available only in limited amounts, bottle-necks may appear, e.g. activities cannot be started on time due to unavailability of resources, or, activities requiring the same resource which is only available one unit at the time must be delayed, etc. These give rise to fresh problems concerning resources.

The various resource problems in project scheduling can be divided broadly into three classes -

i) Time/cost trade off,

ii) Resource Levelling, and,

iii) Resource Allocation.

Time-cost trade-off problem may appear when there are no constraints placed on the availability of

activity direct costs, which include the costs of the material, labour etc. required to perform the activity, and the indirect costs which are made up of the project overheads. The problem then consists of determining the schedules that reduce the project duration time with a minimum increase in the project direct costs, by buying time along the critical path(s) where it can be obtained at least cost, with the use of additional resources.

The time-cost trade-off procedures implicitly assume that the addition of resources causes no conflicts. It is possible, however, that one aims, given a total project completion time, to level the various resource requirements over time. The resource levelling problem occurs when sufficient resources are available for the completion of the project, but one tries to keep the resource usage as much as possible to a constant rate. This problem forms the crux of this thesis.

When total resource usage is limited to a given limit, the objective may be to allocate different resources to the activities in such a way that the project duration is minimized. This is known as the Resource Allocation problem.

Almost all projects employ multiple resources for the execution of their constituent activities. Most heuristic resource levelling attempts in this field

were limited to one or two resources only. The aim of this thesis is to find a set of generalized heuristic policies for this problem.

1.2 Some Significant Aspects of the Problem

Most project networks are made-up of activities which employ multiple resources, which in turn, are limited to some degree. There are considerable fluctuations of the resource requirement during the project period. Pronounced bunching up of resource demands at various points along the project length is decidedly undesirable. The recurrent hiring and lay off of resources on a short term-basis is troublesome, inefficient, and scarcely conducive to attracting and keeping high quality resources - especially skilled personnel. New people on a job are not as efficient as they will be after they become familiar with the intricacies of the job. There is a 'learning curve phenomenon' in the sense that a crew's production goes up and its labour costs go down if kept repeatedly on a job. Then, too, there is the practical consideration that when resources are laid off for a few days, they may be difficult or impossible to replace. Again, there is the aim of the management to keep the hired resources actively engaged as much of the time as possible. All these aspects have to be considered when development of a resource levelling model is in progress.

The problem that arise in resource levelling are : -

i) How to avoid unwanted peaks and valleys in resource levels,

ii) How to resolve levelling conflicts between various resources, and,

iii) How to minimize the resource associated costs of the schedule.

1.3 Scope of this work :

This thesis is an attempt at evolving a set of heuristic policies for the levelling of resources in the networks of multi-resource projects. Here the emphasis for levelling is on the minimization of the resource-associated cost of schedule as a whole rather than on the attainment of the individual minimum levels of resources. Hence, the stress is on the levelling of the costlier resources at the expense of the cheaper ones, which is pardonable, if by doing so a simultaneous reduction in the total resource cost of the project is obtained.

The next chapter is a brief account of the work reported in the literature. In the third chapter, a detailed description of the proposed heuristics is presented. The last chapter contains a discussion of the results obtained by the application of the proposed policies to a set of problems.

CHAPTER 2

LITERATURE REVIEW

2.1 Heuristic Programming in the Resource Scheduling Problem

The conventional scheduling procedures, based on PERT and CPM, implicitly assume that resources required for the activities are unlimited in supply. Whilst this assumption of unlimited resources may be justified in some cases, most project managers are faced with the problem of relatively fixed manpower availabilities, a limited number of machines, budget constraints etc. Jobs occurring on parallel paths through the network may compete for the same scarce resources. If one considers scheduling with resource restriction, it may result in a radical change in the final schedule as compared to the unlimited resource model.

Scheduling projects with limited resources is a type of problem that mathematicians refer to as a large combinatorial problem. Analytical techniques are computationally impractical for most real-life problems of this kind. In recent years a good deal of work has been done in the development of heuristic programs for solving large combinatorial problems.

"A heuristic is a guide or method of reducing search in a problem solving situation. It can be called as an aid to the discovery of a solution. While heuristics may not lead to the optimal solutions in each case, experience over a period of time has proved their general usefulness in finding good solutions to recurring problems, with a minimum of effort. A set of such rules for solving a particular problem is termed a heuristic program."

Project scheduling problems are of three types, viz., time-cost trade-off, resource levelling, and resource allocation. This dissertation is concerned with resource levelling only.

A number of heuristic programs have been developed for the resource levelling problem. Some of these feature in the literature; others, for proprietary reasons, have been kept as a secret. A brief discussion of the techniques available in the literature follows.

2.2 Resource Levelling Programs

The problem of resource levelling arises when there are considerable fluctuations of resource requirement along the length of the project. Such fluctuations are undesirable as they result in frequent hiring and layoff of resources. In case of manpower, such a policy

is detrimental to the employee morale. The economic considerations also perforce the management to look for levelled resource requirements.

One of the earliest works in Resource Levelling was a systematic procedure developed by Burgess and Killebrew [1] in 1962. This method utilized a measure of effectiveness given by the sum of squares of the resource requirements for each period in the project schedule. It was shown that while the sum of the periodic resource requirements over the project was constant for all complete schedules, the sum of the squares of the periodic requirements reduced as the resource peaks were clipped to fill in the valleys. Further this sum reached a minimum for a schedule in which the resource requirement was level, or as nearly level as could be obtained for that project. The model starts with the activities at their earliest start time. It provides for the shifting of activities towards their latest start time only. This program is credited with imparting some mathematical meaning to levelling. This model does not prescribe any criterion for the resolution of conflicts in multi-resource levelling. Hence its applicability, in its present form is confined to simple single or two-resource networks only. Also it requires the activities to be shifted in accordance with the order for precedence, which may leave many other possible schedules unexplored.

A resource levelling program called Multi-shop, Multi-ship (MS^2) Workload Smoothing program, designed to smooth the manpower requirements in the naval ship yards, was presented by Levy, Thompson and Weist [2] in 1962. This employs the critical path method of analysing the jobs to be done on a given project (ship) and begins by assigning all jobs at their earliest start time. Suitable jobs occuring on days with the peak work-load are picked randomly and shifted beyond that day in order to reduce the peak. The program consists of two segments. The first smooths the workload on all the resources simultaneously. The second segment performs further smoothing on individual resources, starting with the most expensive one. The model is capable of handling many projects simultaneously.

A slightly different version of the procedure proposed by Levy et. al. was presented by Wilson [3], designed to give the minimum level of a resource required to complete a project by the specified date. Instead of random choice of activities for shifting, as in Levy's model, Wilson incorporates a dynamic programming scheme at each iteration to determine the feasible combinations of activities which can be shifted to achieve a sizable reduction in the value of the resource peak. However,

he makes the simplifying assumption that each activity requires one unit of the same type of resource.

Dewitte's [4] computerized manpower levelling technique was published in 1964. It is designed to minimize manpower fluctuations by adjusting the start time of project activities having slack. However, the measure of effectiveness of minimization is the absolute magnitude of manpower fluctuations from a calculated project mean level of resource usage. Basically, the method consists of partitioning the resource profile into specially derived intervals of time and then sequentially levelling in each interval, revising the early start times of the activities where necessary.

Many analytical procedures have also been suggested for the problem. Kenneth Baker [5] and Dilip Guha [6] have suggested techniques which employ the concepts of Integer and Zero-One programming. Petrovic [7] reviews the optimization of the resource levelling process as a Dynamic Programming problem. However, most of these analytical techniques are of academic importance as they can be suitably applied to small-sized problems only.

The study of the published literature shows remarkable paucity of work in the field of multiple-resource levelling. In case of the analytical procedures evolved, dimensionality becomes an unmanageable

problem when they are applied to large real-life projects. The heuristic programs suggested by Burgess and Killebrew, Levy et.al, Wilson, and Dewitte are suited to networks constituting of activities employing single resource only. The absence of a heuristic method which could effectively level the periodic requirements of a number of resources simultaneously in multiple-resource networks led to this work.

CHAPTER 3

PROBLEM FORMULATION AND THE PROPOSED POLICIES

3.1 Reviewing the Problem

The term Resource Levelling in a project network means smoothing the resource demand over the length of the project. In case of manpower a stable resource level usually results in good employee morale, fine working environment, and excellent public relations.

Viewed on a broad perspective, all management decisions in a project have the objective of reducing costs and increasing profits. The management recognizes resource levelling as the problem of minimization of the costs associated with resources. The available heuristic methods are capable of handling project networks with activities utilizing single resource only. Real life projects usually have activities which need multiple-resources for their execution. Levelling in such cases would entail simultaneous cost minimization for all the resources. However, it should be clearly understood that cost minimization is used as a tool to attain stable resource levels. A set of heuristic policies using the cost minimization concept for multi-resource levelling is presented here.

3.2 Resource Levelling Policies

The availability of resources at a project site largely governs the levelling policy to be pursued by its management. If the resources are easily available, the management would prefer to hire and fire them as and when required. Whereas, in case of restricted availability of resources and/or high hiring and firing costs, it may be economical to employ the resources for the entire duration of the project. Keeping in mind the availability conditions for the resources and the resource hiring and firing costs, the following three types of resource levelling policies can be formulated :

1. Peak levelling policy :

In cases where the resource availability is limited and/or the hiring and firing costs are prohibitive, the management of a project finds it advisable to employ resources at fixed peak levels throughout the project. Then the objective of the levelling would be to keep the peak values ofthe various resources as low as possible. This policy finds a wide application in projects which employ skilled personnel and special equipment, and/or are executed in remote areas, e.g., the construction of a dam.

2 Minimum hiring-firing policy :

When the resources are easily available on demand and the hiring-firing costs are low, the

management of a project likes to recruit resources in accordance with the needs of the project at a particular time. This policy is usually applied to projects which employ ordinary equipment, daily wage workers etc.

3. Mixed Policy :

As the name suggests, this is a combination of the first two policies. The projects which follow this policy maintain two categories of employees, i.e., full-time and part-time, in each type of resource. The full-time employees are maintained at fixed levels through out the length of the project. They are supplemented by the part-time employees as and when the need arises. This policy is followed by some large construction organisations which have their own specialized full-time employees under each type of resource. They are supplemented by hired part-time employees when needed in the project.

A heuristic resource levelling model which can be used for all these policies will be presented later in thischapter.

3.3 Costs Associated with Resources

Before going into the actual formulation of the resource levelling problem, a discussion of the principal costs which contribute to the objective function is warranted.

In this thesis the following costs will be considered for the levelling purposes :

1) The Recurring cost, and,
2) The hiring and firing cost.

The recurring cost considered here will constitute of the payroll and the costs of fringe benefits for men, and the maintenance cost and the depreciation for the equipment. The recurring cost is assumed linear here.

Mathematically,

$$\text{Recurring cost} = C_{r,n} \cdot W_{t,n}$$

where,

$C_{r,n}$ is the recurring cost per unit for the n^{th} resource and

$W_{t,n}$ is the level ofthe n^{th} resource in the t^{th} period.

The hiring and firing costs are those resulting from increasing the size of the resource level by hiring or reducing the size by layoff. Both hiring and layoff costs increase with the size of level variation for a resource. The factors that contribute to these costs are training, relocation, compensation etc. Holt et. al [8] have suggested a quadratic approximation for these costs. The same is assumed in this work also.

Mathematically,

$$\text{Hiring/firing costs} = C_{hf,n} \, (W_{t,n} - W_{t-1,n})^2$$

where,

$C_{hf,n}$ is the cost of hiring-firing per (unit of resource)^2 for n^{th} resource

$W_{t,n}$ is the resource level at t^{th} time period of the project for n^{th} resource.

3.4 Objective Function and the Constraints

The total resource associated cost of a schedule for a particular time-period would be the sum of the recurring and the hiring-firing costs for that period. For a levelled resource requirement over the project length this cost should be minimized.

The objective function for levelling can be written as,

$$\text{minimize} \quad C_T = \sum_{n=1}^{N} \sum_{t=1}^{T} C_{r,n} \cdot W_{t,n} + \sum_{n=1}^{N} \sum_{t=1}^{T} C_{hf,n} \cdot (W_{t,n} - W_{t-1,n})^2$$

where,

C_T is the total resource cost of the schedule.

$C_{r,n}$ is the recurring cost per unit resource for the n^{th} resource.

$C_{hf,n}$ is the hiring or firing cost per (unit)^2 for the n^{th} resource.

$W_{t,n}$ is the level of n^{th} resource in the t^{th} period.

T is the total time for the schedule, and

N is the number of resources in the project.

Here, the expression,

$\sum_{n=1}^{N} \sum_{t=1}^{T} C_{r,n} \; W_{t,n}$ represents the total recurring cost for the schedule,

and, $\sum_{n=1}^{N} \sum_{t=1}^{T} C_{hf,n} \; (W_{t,n} - W_{t-1,n})^2$ is the total hiring-firing cost for the schedule.

The constraints on the objective function are that the resource levels are controlled by the starting and finishing time of the activities, and these are restrained to vary between the limits specified by the management of the project. The problem is to find that schedule of activities which minimizes the objective function.

As mentioned in an earlier section of this chapter, the objectives are different for each of the three levelling policies discussed. The objective function stated above shall be modified to suit the policy being considered. These shall be discussed later in the chapter.

3.5 The Proposed Levelling Model

The resource levelling model proposed for the three levelling policies discussed in this thesis consists of the following heuristics :

1) Calculation of the CPM schedule,
2) Forward Search routine,
3) Back-tracking routine, and,
4) Crashing of non-critical activities routine.

The first heuristic calculates the usual parameters such as the earliest and the latest start times, and the slack values for the various activities of the network. It also gives the total length of the project.

The forward search routine is the main operational heuristic of the model. This allows the activities to shift towards their latest start times, in the process improving upon the objective function for levelling. The routine starts with all the activities of the network scheduled to start at their earliest start times. The value of the objective function for this schedule acts as its initial value. The rescheduling of the various activities is attempted to improve the objective function. A unique feature of this routine is that the sequence in which activities are tried for rescheduling

can be varied with the help of an inbuilt random selection routine. This helps in the generation of several alternate schedules for the network of which the one giving the minimum value of the objective function is selected.

The third heuristic provides for the shifting of activities towards their earliest start times. This back-tracking helps in the relocation of some activities which results in a lower value of the objective function. This routine is primarily an accessory to the forward search routine because it searches for any possible good schedule which was bypassed while the second heuristic was being executed.

The Crashing routine is a novel feature of this model. It was observed that in many networks the slack values are insufficient to facilitate good resource levelling. It was found that the crashing of some of the non-critical activities of the network could result in a levelled resource pattern. This inspired us to include the present crashing routine in our model. The crashing cost is assumed linear in this model. It is further assumed that the activities are independent, in the sense that buying time on one activity does not affect in any way the availability, cost, or need to buy time on some other activity. The routine in itself is quite simple in nature. The activity selected for

crashing is rescheduled to start at a position which brings about the maximum reduction in the objective function.

A stepwise illustration of the levelling method is as follows. A Flow-Chart for this is given in Appendix B.

STEP 1.

The earliest start time, the latest start time, and the slack values for all the activities of the network are calculated.

STEP 2.

The Earliest Start Schedule (ESS) for the network is prepared and the corresponding value of the objective function is calculated. This serves as its initial value.

STEP 3.

A list of activities which have no follower activities, or whose follower activities have already been scheduled, is prepared. It is in the order of descending activity numbers. This list is called as the available activity set (AAS). Each member of the ASS has equal probability of being scheduled.

STEP 4.

An activity is selected randomly from the AAS. The probability of selection of the x^{th} activity

in the list is given by,

$$\frac{P\,(1 - P)^{(x-1)}}{1 - (1 - P)^{n}}$$

where,

P is an input probability value, and

n is the total number of activities in the AAS.

STEP 5.

The selected activity is rescheduled to give the lowest value of the objective function. If more than one schedule gives the same value of the objective function, the activity is scheduled as late as possible to get the maximum possible slack in all preceding activities.

STEP 6.

The AAS is updated to include any activities rendered eligible by shifting the previously selected activity.

STEP 7.

Keeping the previously shifted activity fixed, STEPS 4 to 6 are repeated with the next selected candidate.

STEP 8.

STEP 7 is continued until all the activities in the network have been considered; this completes the first rescheduling cycle.

STEP 9.

Additional rescheduling cycles are carried out by repeating STEPS 3 through 8 until no further reduction in the objective function is possible, noting that only movement of an activity towards its latest start value is permissible under this scheme.

STEP 10.

STEPS 3 through 9 are repeated as many times as possible. Because of the random elements in the program, different schedules result from each application of the method. The schedule having the lowest value of objective function is selected as the final one.

STEP 11.

Starting with the last activity, it is seen whether any improvement in the objective function is obtained by shifting it towards its earliest start value. The new schedule is noted.

STEP 12.

STEP 11 is repeated with the other activities of the network till the entire list is exhausted. This completes the first rescheduling cycle of the Backtracking heuristic.

STEP 13.

Additional rescheduling cycles are carried out by repeating STEP 11 and 12 until no further reduction in the objective function is noted.

STEP 14.

The peak in the resource pattern of the first resource in the resource list is located.

STEP 15.

A list is prepared of the activities that contribute to this peak and can be crashed as well.

STEP 16.

If the crashing of any of the activities in the list is able to reduce the peak and the objective function simultaneously, then it is crashed.

STEP 17.

New resource patterns are generated and peaks located.

STEP 18.

STEPS 13 through 17 are carried out repeatedly until the peak in the first resource and the objective function cannot be reduced further.

STEP 19.

The next resource in the list is selected and STEPS 13 through 18 are carried out for it.

STEP 20.

STEP 19 is repeated till all the resources have been considered.

The levelling methods for the three policies which will be discussed in next section are minor

variations of that presented in the above steps. The differing steps shall be discussed only in the individual discussion of the levelling policies. In the following sections the three levelling policies tested for the multiple-resource levelling problem are described.

3.6 POLICY 1 : Peak-levelling Policy

In this policy, the peaks in the demand patterns of the various resources are minimized. The project management employs resources at these peak values through the length of the project.

3.6.1 Objective function and the levelling method

The levelling is achieved here by minimizing the sum of the recurring and the hiring-firing costs. The hiring and firing costs are incurred only twice in this policy, i.e., at the beginning and at the end of the project. The recurring costs are borne at a constant rate throughout the project.

Mathematically, the objective function is,

$$\text{minimize} \quad C_T = \sum_{n=1}^{N} C_{r,n} W_{p,n} T + \sum_{n=1}^{N} C_{h,n} W_{p,n}^{2} + \sum_{n=1}^{N} C_{f,n} W_{p,n}^{2}$$

where,

$W_{p,n}$ is the peak value for n^{th} resource,

T is the total length of the project,

$C_{h,n}$ is the hiring cost per square unit for n^{th} resource, and

$C_{f,n}$ is the firing cost per square unit for n^{th} resource.

The levelling method for this policy is identical to the general case described in Section 3.5.

3.7 POLICY 2 : Minimum Hiring-Firing Policy

This policy is an extension of the one suggested by Burgess and Killebrew [1] . Levelling for individual resources in this case is obtained by minimizing the corresponding sum of squares of the daily resource requirements. This would mean employment of resources in accordance with the periodic requirements. If the shifting of an activity results in the reduction of the sum of squares for one resource and in an increase for others, the conflict is resolved by studying the effect of the shifting on the objective function, which shall be described now.

3.7.1 Objective function and the levelling method

The recurring costs remain the same for all schedules in case of this policy. The daily hiring and firing of resources in this policy indicates that the

resources will be levelled when the sum of squares of their daily resource requirements is minimized. The sum of squares of resource level represents the hiring and firing costs also. Thus the objective function in this policy is the minimization of the sum of products of the hiring-firing costs and the sum of squares of daily resource levels for all resources.

Mathematically, the objective function is expressed as,

$$\text{minimize} \quad C_T = \sum_{n=1}^{N} \sum_{t=1}^{T} C_{h,n} W_{t,n}^2 + \sum_{n=1}^{N} \sum_{t=1}^{T} C_{f,n} W_{t,n}^2$$

The levelling method for this policy is the general model discussed in Section 3.5.

3.8 <u>POLICY 3</u> : <u>Mixed Policy</u>

In this policy, the resource requirement is fulfilled by two categories of the same resource, viz., Full-time employees and Part-time employees. The Full-time employee level, once selected, is maintained fixed for the problem. The Part-time employee component of resource is hired and fired daily in accordance with the resource requirement for that day. The cost contributed by the latter component to the total resource cost of the schedule can only be minimized in this policy. It is assumed here that the cost coefficients, viz., $C_{r,n}$,

$C_{h,n}$ and $C_{f,n}$ are the same for both Full-time and Part-time employees.

3.8.1 Objective function and Levelling method

The total resource cost in this policy is contributed by the fixed cost, due to Full-time employees, and the variable cost, due to Part-time employees. The variable cost constitutes of recurring and hiring-firing costs. The recurring cost is reduced when the total number of part-time employees for a resource are reduced. The hiring-firing cost is reduced when the sum of squares of the daily resource requirements for a resource are reduced. This levelling policy attempts simultaneous minimization of both of these costs.

Mathematically, the objective function is,

$$\text{minimize} \quad C_T = \sum_{n=1}^{N} \sum_{t=1}^{T} C_{r,n} (W_{ot,n} - W_{f,n})$$

$$+ \sum_{n=1}^{N} \sum_{t=1}^{T} C_{h,n} (W_{ot,n} - W_{f,n})^2$$

$$+ \sum_{n=1}^{N} \sum_{t=1}^{T} C_{f,n} (W_{ot,n} - W_{f,n})^2$$

where, $W_{f,n}$ is the full-time resource level for n^{th} resource,

and, $W_{ot,n}$ is the total resource level for n^{th} resource on the t^{th} period.

A constraint is that if $(W_{ot,n} - W_{f,n}) < 0$, then $(W_{ot,n} - W_{f,n})$ is taken equal to zero.

The levelling method for this policy is a modified version of that presented in Section 3.5. The second step of the general model is preceded by an additional step in which the Full-time employee levels for the various resources of the network are calculated. This is obtained by dividing the total requirement for a resource in the project by the length of the project. Then the ESS is determined and the objective function is calculated. STEPS 3 to 13 are similar to those of the general model.

The crashing heuristic is slightly modified for this policy. The resources are listed in the order of decreasing recurring cost per unit. The first resource of this list of resources is considered for crashing. It is found out whether the requirements for this resource exceeds the Full-time employee level on any period of the project. The periods at which Part-time employees are needed are tested for crashing. It is seen whether the crashing of a non-critical activity can improve the objective function. This is carried out on all days when Part-time employees are used.

Similar application of the crashing routine is tried on all resources.

These policies were tested for a set of problems. The results are given in the next chapter.

A Computer Program for the general levelling model is given in Appendix C.

CHAPTER 4

RESULTS AND DISCUSSION

The heuristic model proposed in the previous chapter was programmed in Fortran IV language and a set of problems was solved on IBM 7044. Some networks were taken from books on network analysis, while others were constructed randomly. The paucity of live data led to the assignment of random values to the problems. The data for problems tested are compiled in Appendix A.

The results are given in Tables 1 through 7. Individual discussion of the results shall follow later.

The Forward Search heuristic of the model has a random search routine which is capable of generating different schedules on each of its applications to a problem. In this study a maximum of twenty such iterations were allowed for each network. Another condition imposed was that the search was terminated when a schedule could not be improved upon in ten successive applications of the heuristic. These conditions can be easily modified if a more comprehensive search is desired by the user.

TABLE 1

Value of Peaks, sum of squares, and Number of Part-time employees for different schedules.

Problem No.	No of Activities	No. of Resources	Length of project (days)	Average level of Resources		POLICY 1			
						ESS		FS	
						PEAKS		PEAKS	
				1	2	1	2	1	2
1	6	2	10	4	7	6	10	4	7
2	8	2	10	5	7	7	9	5	7
3	5	2	10	3	5	4	7	3	5
4	6	2	8	2	3	3	5	2	3
5	8	2	10	4	6	7	10	6	9
6	8	2	13	2	6	4	10	4	8
7	22	2	27	4	11	8	20	6	16
8	13	2	25	3	6	5	17	6	8
9	14	2	39	4	7	15	16	10	12
10	15	2	24	8	17	22	36	13	21

Table 1 continued on next page

NOTE:

ESS is Earliest Start Schedule

FS is Final Schedule

TABLE 1 (Continued)

Problem No	POLICY 2								POLICY 3							
	ESS				FS				ESS				FS			
	PEAKS		SS		PEAKS		SS		PEAKS		PTE		PEAKS		PTE	
	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1	6	10	202	568	4	7	160	490	6	10	9	12	4	7	0	0
2	7	9	302	566	5	7	250	490	7	9	10	12	5	7	0	0
3	4	7	108	274	3	5	90	250	4	7	6	6	3	5	0	0
4	3	5	34	80	2	3	32	72	3	5	1	2	2	3	0	0
5	7	10	206	424	6	9	170	380	7	10	9	11	6	9	4	6
6	4	10	103	644	4	8	99	564	4	10	11	20	4	10	10	20
7	8	20	532	4238	8	14	428	3478	8	20	36	69	8	20	35	59
8	5	17	397	1257	5	12	347	1031	5	17	30	46	5	12	21	36
9	15	16	1388	2277	10	14	1158	2197	15	16	75	69	10	16	69	65
10	22	36	2382	9878	12	25	1870	7500	22	36	46	106	13	24	28	56

NOTE :

SS is Sum of Squares

PTE is the total number of Part-time Employees

TABLE 2

Reduction Obtained in the value of the Objective Function .

Problem No.	POLICY 1 Objective Function in Rs.			POLICY 2 Objective Function in Rs.			POLICY 3 Objective Function in Rs.		
	ESS	FS	% Reduction	ESS	FS	% Reduction	ESS	FS	% Reduction
1	5760	3756	34.7	4292	3560	17.1	960	0	100.0
2	13290	9490	28.6	11700	9900	15.4	2140	0	100.0
3	3756	2690	28.4	2176	1900	12.7	528	0	100.0
4	3090	1872	39.5	980	896	8.56	172	0	100.0
5	6990	6015	13.92	4180	3600	13.9	1090	480	56.0
6	15040	13560	9.85	4765	4305	9.46	3255	3060	6.0
7	46792	36128	22.8	25446	20814	17.9	10697	9269	11.6
8	30906	23616	23.6	8998	7594	15.6	6848	4562	33.5
9	131364	91416	30.6	37149	33669	9.6	23043	19875	13.7
10	68564	38369	43.9	51422	39350	23.4	17992	6742	62.5

TABLE 3

Resource demand patterns for Problem No. 8 and Policy 1

Resource / Day	ESS 1	ESS 2	Pre-crashing Schedule 1	Pre-crashing Schedule 2	Final Schedule 1	Final Schedule 2
1	5	6	5	6	6	8
2	5	6	5	6	6	8
3	5	6	5	6	6	8
4	5	6	5	6	6	8
5	5	6	5	6	6	8
6	5	6	5	6	6	8
7	4	17	4	12	2	8
8	4	17	4	12	2	8
9	4	12	4	12	2	8
10	2	8	2	8	2	8
11	5	2	5	7	5	7
12	5	2	5	7	5	7
13	5	3	5	3	5	3
14	5	3	5	3	5	3
15	5	3	5	3	5	3
16	4	3	4	3	4	3
17	5	2	3	1	3	1
18	5	2	3	1	3	1
19	1	7	3	8	3	8
20	1	7	3	8	3	8

Contd. on next page

TABLE 3 (Continued)

Day \ Resource	ESS 1	ESS 2	Pre-crashing Schedule 1	Pre-crashing Schedule 2	Final Schedule 1	Final Schedule 2
21	1	6	1	6	1	6
22	1	6	1	6	1	6
23	0	4	0	4	0	4
24	0	4	0	4	0	4
25	0	1	0	1	0	1

TABLE 4

Resource demand patterns for Problem No. 7 and Policy 2

Resource / Day	ESS 1	ESS 2	Pre-crashing Schedule 1	Pre-crashing Schedule 2	Final Schedule 1	Final Schedule 2
1	8	10	4	6	5	10
2	8	10	4	6	5	10
3	7	14	3	10	4	14
4	7	14	5	10	4	6
5	5	20	3	16	2	12
6	5	20	3	16	2	12
7	3	18	3	12	3	12
8	3	18	3	12	3	12
9	3	18	2	12	2	12
10	2	20	2	12	2	12
11	3	18	3	12	3	12
12	4	19	3	12	3	12
13	6	11	5	10	5	10
14	6	11	5	10	5	10
15	6	11	4	8	4	8
16	7	5	8	9	8	9
17	3	9	6	13	6	13
18	2	6	6	13	6	13
19	2	6	4	13	4	13
20	3	6	4	13	4	13

Continued on next page

TABLE 4 (Continued)

Day \ Resource	ESS		Pre-crashing Schedule		Final Schedule	
	1	2	1	2	1	2
21	3	6	4	13	4	13
22	2	6	2	12	2	12
23	2	6	2	12	2	12
24	0	6	3	12	3	12
25	0	6	3	12	3	12
26	0	4	3	8	3	8
27	0	4	3	8	3	8

TABLE 5

Resource demand patterns for Problem No. 9 and Policy 3

Resource / Day	ESS		Pre-crashing Schedule		Final Schedule	
	1	2	1	2	1	2
1	5	14	5	14	5	8
2	5	14	5	14	5	8
3	5	6	5	6	5	8
4	5	6	5	6	5	8
5	5	6	5	6	5	8
6	5	6	5	6	5	8
7	5	6	5	6	5	8
8	9	14	9	14	9	16
9	7	8	7	8	7	8
10	7	8	7	8	7	8
11	7	8	7	8	7	8
12	7	8	7	8	7	8
13	15	16	10	6	10	6
14	10	6	10	6	10	6
15	10	6	10	6	10	6
16	10	6	10	6	10	6
17	8	8	8	8	8	8
18	8	8	8	8	8	8
19	8	8	8	8	8	8
20	8	8	8	8	8	8

Continued on next page

TABLE 5 (Continued)

Day \ Resource	ESS 1	ESS 2	Pre-crashing Schedule 1	Pre-crashing Schedule 2	Final Schedule 1	Final Schedule 2
21	4	2	9	12	9	12
22	7	8	4	8	4	8
23	7	8	4	8	4	8
24	4	8	4	8	4	8
25	4	8	4	8	4	8
26	0	8	3	8	3	8
27	0	8	3	8	3	8
28	0	8	0	8	0	8
29	0	8	0	8	0	8
30	0	9	0	9	0	9
31	1	2	1	2	1	2
32	1	2	1	2	1	2
33	1	2	1	2	1	2
34	1	2	1	2	1	2
35	1	2	1	2	1	2
36	1	2	1	2	1	2
37	1	2	1	2	1	2
38	1	2	1	2	1	2
39	1	2	1	2	1	2

4.1 Discussion of some Individual Results

The Tables 1 and 2 are summarized versions of the results for the set of problems. To illustrate the working of the heuristics of the levelling model, the comprehensive results of an individual problem shall be discussed for each policy.

Policy 1 :

Consider the Eighth problem from Table 1. This consists of 13 activities, employs two resources, and is 25 days in length. The resource demands for the earliest start schedule (ESS)are given in Table 3. It can be seen from Table 3 that peak values for the first and second resources are 5 and 17 respectively. The value of the objective function is Rs. 30906 for the ESS.

The Forward Search heuristic was executed eleven times in this case. The peak value of the first resource remained unchanged at 5, while that for the second resource reduced to 12. The no reduction in the peak of the first resource can be attributed to the presence of as many as thirteen peak days in the ESS resource demand pattern (refer Table 3). The value of the objective function reduced from Rs. 30906 to Rs. 25326.

The application of the Back-tracking heuristic did not result in any further reduction in peaks and the objective function.

The crashing heuristic was used next. It was found ineffective for the first resource as the crashing of the eligible activities in this case leads to the establishment of worse peaks elsewhere in the project. For the second resource the peak value of 12 occurs on the seventh, eighth, and ninth days of the schedule. The second, third, and fourth activities contribute to these peaks. The third activity, being a critical one, cannot be crashed. The second activity was crashed. This resulted in the peaks of 6 and 8 for the two resources. The objective function reduced from Rs. 25326 to Rs. 23616. The increase in the peak value of the first resource is excusable as the overall cost of the new schedule is lesser than that of the earlier schedule. Further application of the crashing heuristic did not result in any improvement in the objective function. The final resource demand patterns are shown in Table 3.

Policy 2

The seventh problem from Table 1 is considered for this policy. It has 22 activities, uses two resources, and the length is 27 days. The resource demand patterns for the ESS are given in Table 4. For the ESS, the value of the sum of squares for the first and second resources are 532 and 4238 respectively. The value of the objective function is Rs. 25446 for this schedule.

The Forward Search Routine underwent twelve iterations for this problem. In the process, the sum of squares for the first resource reduced from 532 to 422; and for the second resource it reduced from 4238 to 3578. The objective function improved from Rs. 25446 to Rs. 21266.

The application of the Back-tracking heuristic was successful for the eighth, ninth, and eleventh activities. Their rescheduling reduced the sum of squares for the second resource from 3578 to 3542. However, the sum of squares for the first resource remained unaffected at 422. The new value of the objective function was Rs. 21086.

The Crashing heuristic was unsuccessful for the first resource. For the second resource it was found that the fifth activity of the network could be crashed productively. The effects of this crashing were that the objective function reduced from Rs. 21086 to Rs. 20814, the sum of squares for the first resource increased from 422 to 428 and that for the second resource decreased from 3542 to 3478. As the objective function had reduced, the new schedule was accepted as better than the earlier one. No further crashing was found possible. The resource demand patterns for the final schedule are shown in Table 4.

POLICY 3

The results obtained for the ninth problem in Table 1 will be used for illustration in this policy, This problem has 14 activities, uses two resources, and is 39 days long. The resource demand patterns for the ESS of this problem are shown in Table 5. The number of total part-time employees required for the first and second resources for ESS is 75 and 69 respectively. The value of the objective function for this schedule is Rs. 23043.

The application of the Forward Search heuristic reduced the value of the objective function from Rs. 23043 to Rs. 20451 in eleven iterations. In this process the total number of part-time employees reduced from 75 to 69 for the first resource and from 69 to 65 for the second resource.

The use of the Back-tracking heuristic did not result in any further reduction in the value of the objective function in this case.

The Crashing heuristic was unsuccessful in the case of the first resource. The application of the heuristic to the second type of resource proved more fruitful. The fourth activity was crashed. The total number of Part-time employees remained unchanged for

both the resources but the objective function reduced from Rs. 20451 to Rs. 19875. The new schedule was adopted as the final one. The resource demand patterns for the final schedule are given in Table 5.

4.2 General Discussion of the Results

The problems 1,2,3 and 4 of Table 1, were deliberately constructed in such a way that their optimal levels could be determined manually. These problems were used to test the ability of the three proposed levelling policies in achieving the optimal levels for simple networks. It was found that the policies did converge to the optimal solution for all these problems.

The reduction in the value of the objective function was sizable for most of the problems (Refer Table 2). For the Peak-levelling policy it was in the range of 10% to 44%. This high reduction can be attributed to the reduction in the cost of idle resource payoffs as a levelled schedule is obtained. The peaks in the resource patterns also reduced significantly in most cases (as shown in Table 1).

In case of the Second policy, the reduction in the value of the objective function was in the range of 9% to 24%. The sum of squares of daily resource requirements also underwent a sizable reduction in all problems.

The third policy achieved a 100% reduction in the objective function for problems 1,2,3 and 4. This is because the total number of part-time employees for perfectly levelled schedules is zero. The reduction in the value of the objective function for other problems was in the range of 6 % to 62%. The total number of part-time employees also reduced considerably (refer Table 1).

TABLE 6

Time required for solving various problems by Mixed Policy

	PROBLEM NUMBER									
	1	2	3	4	5	6	7	8	9	10
Number of Activities	6	8	5	6	8	8	22	13	14	15
Solution Time in Secs.	2	3	2	1	2	2	20	10	22	25
Number of iterations	11	11	11	11	11	14	11	14	11	17

The time required for solving different sized problems was computed for the third policy. From the results, given in Table 6 , it is apparent that nothing definite can be said about the relationship between the size of a network and the time required to solve it. Conceptually the time taken should increase with the increase in the number of activities. This holds good for some problems. But at the same time there are

instances of contradiction also. There is a strong indication that time increases with the increase in the number of iterations, which is self-explainatory. This, in some ways, explains the observation that some of the smaller networks require more time for solution than the larger ones.

It is apparent from the results in Table 6 that the number of iterations required for solving a particular problem has no obvious relation with its size. An example is the case of problems 1 and 7. They differ in size by 16 activities, but the search for better solution than previous ones is terminated after 11 iterations in both cases.

The policies could not be tested for networks employing more than two resources. However, the model can be used for such cases also.

In conclusion it can be said that this set of heuristic policies gives fairly good results for most networks. The optimality of the results increases with the increase in the computer time.

Suggestions for Future Research

The levelling model and policies suggested in this work were found successful for most networks. However there is much scope for improvement in them. It is recommended that the future investigation should be directed in the following areas :

1. The crashing heuristic of the model may be modified to accommodate cases where the crashing is non-linear in nature. A further addition to the model may be made by allowing splitting of activities. It is felt that these features will improve the model.

2. The policies may be made more broad based by including more cost factors. The objective functions for the second and third policies can be modified to handle different project objectives.

3. The model can easily be extended to handle Repetetive project and Multi-project problems also.

4. The use of Simulation in levelling problems can also be explored. The present model assumes that a single policy is applicable to all the resources in a network. An extension would allow the use of different policies for different resources in the network.

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APPENDIX A

DATA FOR THE SET OF PROBLEMS

TABLE 7

DATA FOR DIFFERENT PROBLEMS

Problem No	Activity No	Start Node for the activity	End Node for the activity	Normal Time duration(days)	Normal Requirement of Resources 1	Normal Requirement of Resources 2	Maximum Requirement of Resources 1	Maximum Requirement of Resources 2
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
1	1	1	2	5	2	3	2	3
	2	1	3	3	1	1	1	1
	3	1	4	3	2	4	2	4
	4	2	5	1	2	3	2	3
	5	3	5	4	3	4	3	4
	6	4	5	7	1	3	1	3
2	1	1	2	2	2	4	2	4
	2	1	3	2	1	1	1	1
	3	1	4	2	2	2	2	2
	4	1	5	5	1	2	1	2
	5	2	6	8	1	2	1	2
	6	3	6	5	3	3	3	3
	7	4	6	3	2	2	2	2
	8	5	6	3	2	3	2	3
3	1	1	2	3	2	2	2	2
	2	1	3	3	1	0	1	0
	3	1	4	10	1	3	1	3
	4	2	4	3	1	2	1	2
	5	3	4	4	2	2	2	2

Continued

TABLE 7 (Contd.)

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
4	1	1	2	3	1	2	1	2
	2	1	3	4	1	1	1	1
	3	2	4	2	1	2	1	2
	4	3	4	1	1	2	1	2
	5	3	5	4	1	1	1	1
	6	4	5	2	1	2	1	2
5	1	1	2	4	1	1	1	1
	2	1	3	2	1	2	1	2
	3	1	4	3	4	6	4	6
	4	2	6	3	2	2	2	2
	5	3	5	2	1	2	1	2
	6	3	6	3	1	1	1	1
	7	4	5	4	2	3	2	3
	8	5	6	3	1	3	1	3
6	1	1	2	5	1	4	1	4
	2	1	3	6	0	4	0	6
	3	1	4	3	3	0	3	0
	4	2	5	2	0	4	0	8
	5	3	7	7	1	2	1	2
	6	4	6	4	2	2	4	4
	7	5	7	1	2	0	2	0
	8	6	7	2	1	4	2	8

Continued

TABLE 7 (Contd.).

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
7	1	1	2	6	2	4	3	6
	2	1	3	4	2	0	4	0
	3	1	4	2	1	2	2	4
	4	1	5	9	2	0	3	0
	5	1	6	6	1	4	2	8
	6	3	7	8	0	6	0	8
	7	4	8	4	0	6	0	8
	8	5	9	2	1	2	2	4
	9	6	10	2	1	0	2	0
	10	7	11	4	2	0	4	0
	11	8	12	9	0	6	0	9
	12	9	13	6	2	3	4	6
	13	10	14	2	1	0	2	0
	14	2	15	4	0	6	0	8
	15	11	16	9	0	4	0	6
	16	13	17	2	1	0	2	0
	17	14	18	6	2	2	3	3
	18	12	19	6	1	0	2	0
	19	15	19	2	0	2	0	3
	20	16	19	2	0	4	0	5
	21	17	19	4	2	0	4	0
	22	18	19	9	0	2	0	3
8	1	1	2	6	2	0	3	0
	2	1	3	9	2	4	3	6
	3	2	3	4	0	6	0	8
	4	1	4	12	1	2	2	4
	5	2	5	2	0	5	0	5
	6	3	5	8	3	0	6	0
	7	4	6	4	1	3	2	6
	8	2	7	9	1	0	3	0
	9	5	7	6	0	4	0	6

Continued

TABLE 7 (Contd.)

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
8	10	6	7	2	2	1	4	2
	11	5	8	4	1	2	2	4
	12	6	8	4	0	1	0	2
	13	7	8	1	0	1	0	1
9	1	1	2	7	3	0	7	0
	2	1	3	2	0	8	0	8
	3	2	3	9	6	4	9	6
	4	1	4	8	0	6	0	8
	5	2	4	5	1	4	1	4
	6	1	5	8	2	0	4	0
	7	4	5	9	4	2	6	3
	8	3	6	4	4	6	8	8
	9	4	6	1	5	10	1	10
	10	5	6	8	0	8	0	8
	11	5	7	4	4	0	8	0
	12	6	7	1	0	9	0	9
	13	5	8	2	3	0	6	0
	14	7	8	9	1	2	3	6
10	1	1	2	5	2	8	2	8
	2	1	3	4	6	9	8	12
	3	2	4	4	5	5	10	10
	4	2	5	2	5	3	10	6
	5	2	6	6	2	10	3	15
	6	3	6	12	3	6	4	8
	7	3	7	2	1	7	1	7
	8	3	8	9	3	5	3	5
	9	4	9	1	2	8	2	8
	10	5	9	6	1	6	2	12
	11	6	9	4	5	3	10	6
	12	7	10	2	4	4	8	8
	13	8	10	3	1	3	3	9
	14	9	11	2	6	9	6	9
	15	10	11	8	2	4	4	8

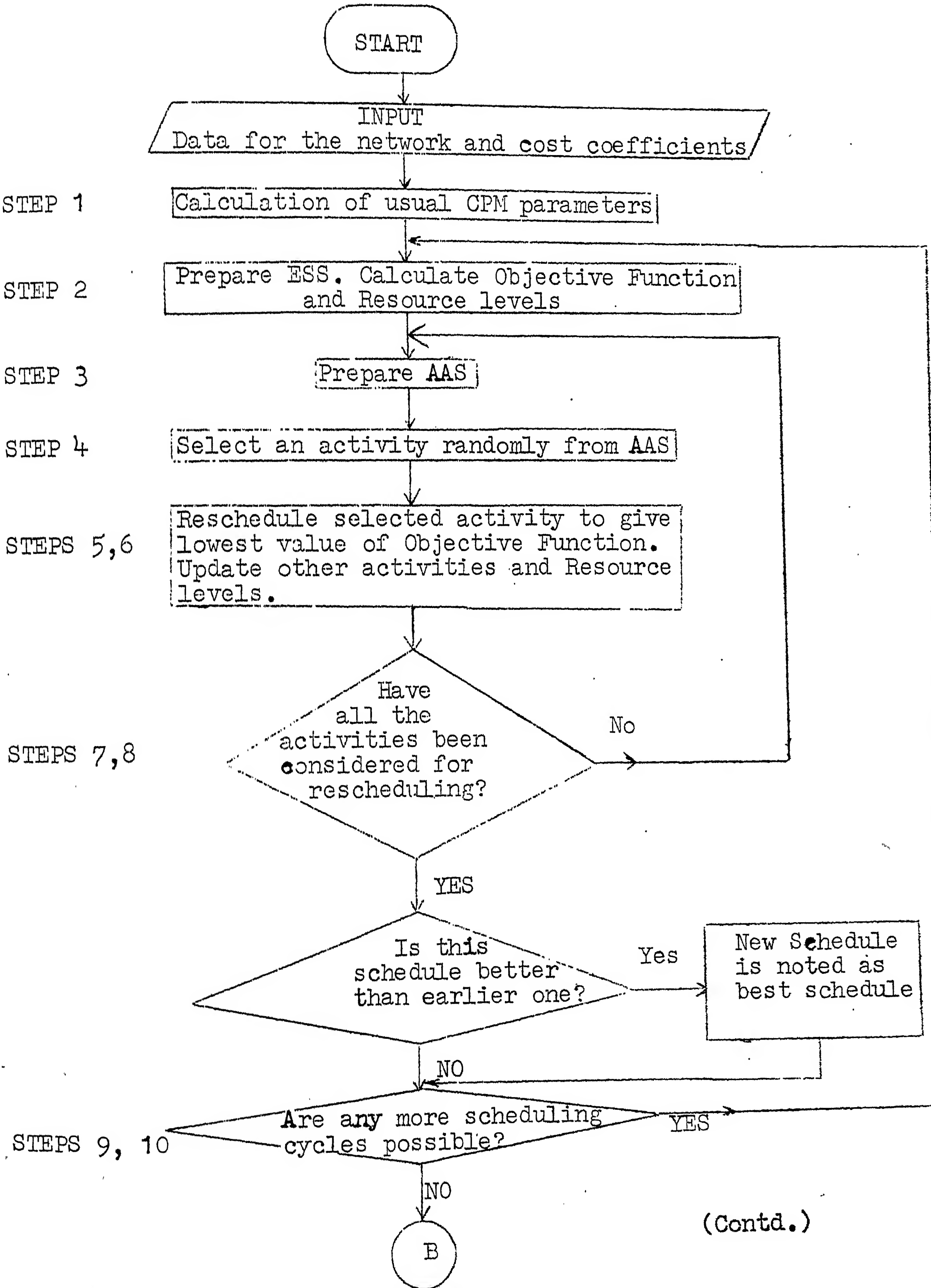
TABLE 8

RECURRING, HIRING AND FIRING COSTS

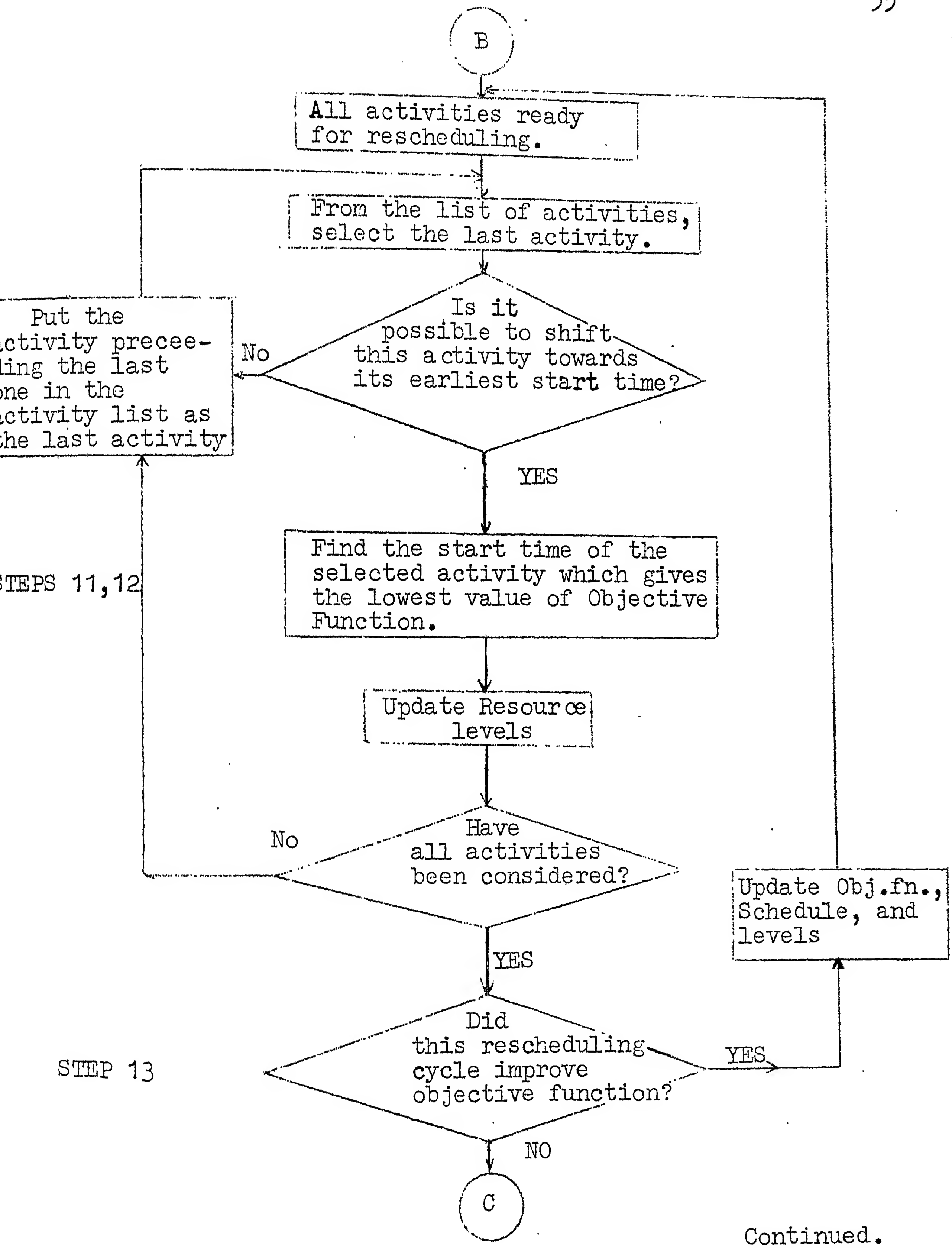
Problem No.	Recurring Cost Rs./Unit		Hiring Cost Rs./Unit^2		Firing Cost Rs./Unit^2	
	1	2	1	2	1	2
1	50	20	5	2	5	2
2	100	50	10	5	10	5
3	50	20	5	2	5	2
4	50	25	5	3	5	2
5	50	40	5	4	5	4
6	150	50	8	3	7	2
7	80	50	4	3	4	2
8	100	40	5	2	5	2
9	120	90	6	5	6	4
10	100	40	3	2	2	2

APPENDIX B

FLOW CHART

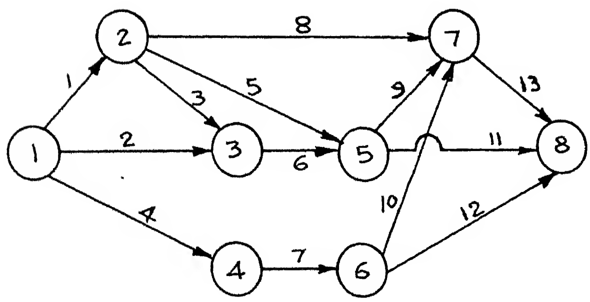


(Contd.)

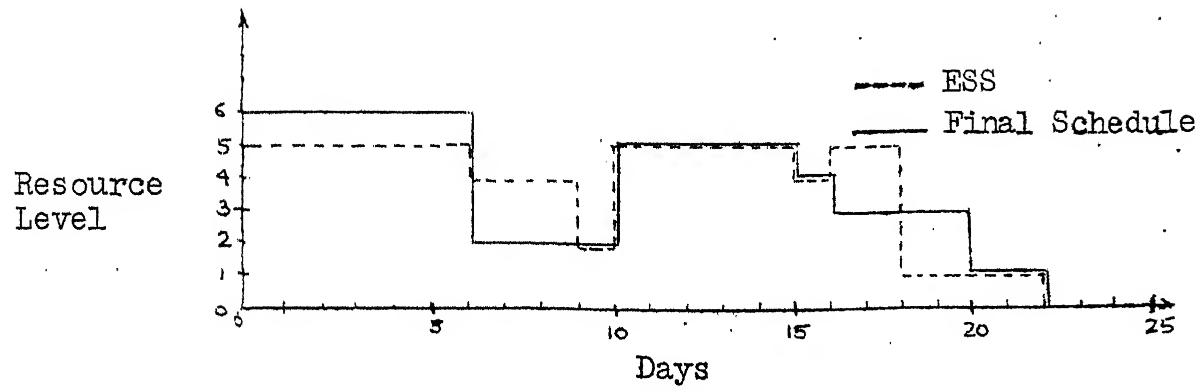

B
All activities ready for rescheduling.
From the list of activities, select the last activity.
Is it possible to shift this activity towards its earliest start time?
No
Put the activity preceeding the last one in the activity list as the last activity
YES
Find the start time of the selected activity which gives the lowest value of Objective Function.
STEPS 11,12
Update Resource levels
Have all activities been considered?
No
YES
Did this rescheduling cycle improve objective function?
STEP 13
YES
Update Obj.fn., Schedule, and levels
NO
C

Continued.

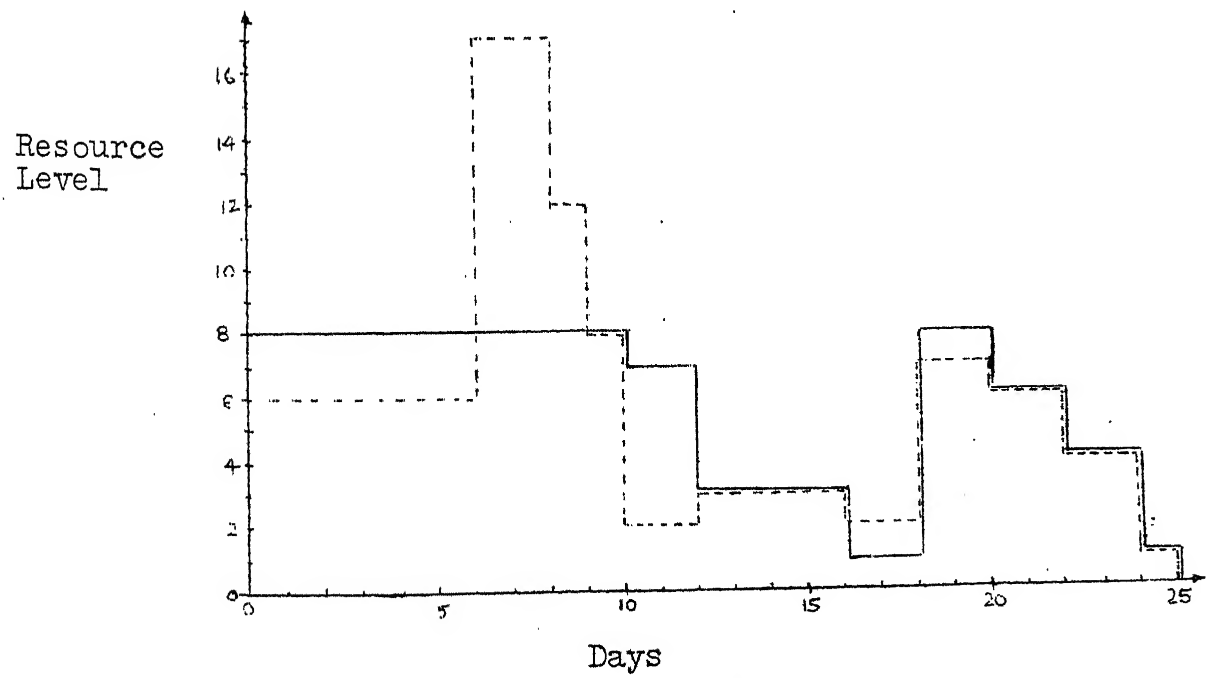
APPENDIX C FIGURES



Network

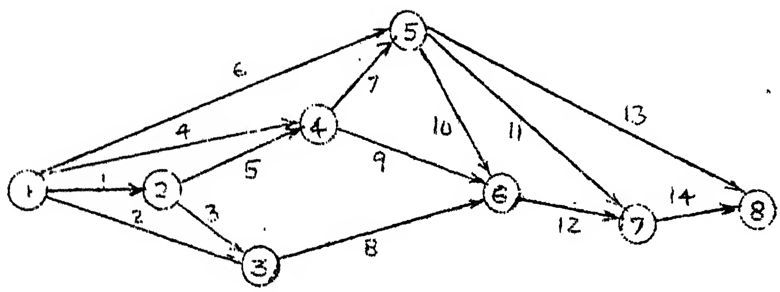


Resource 1

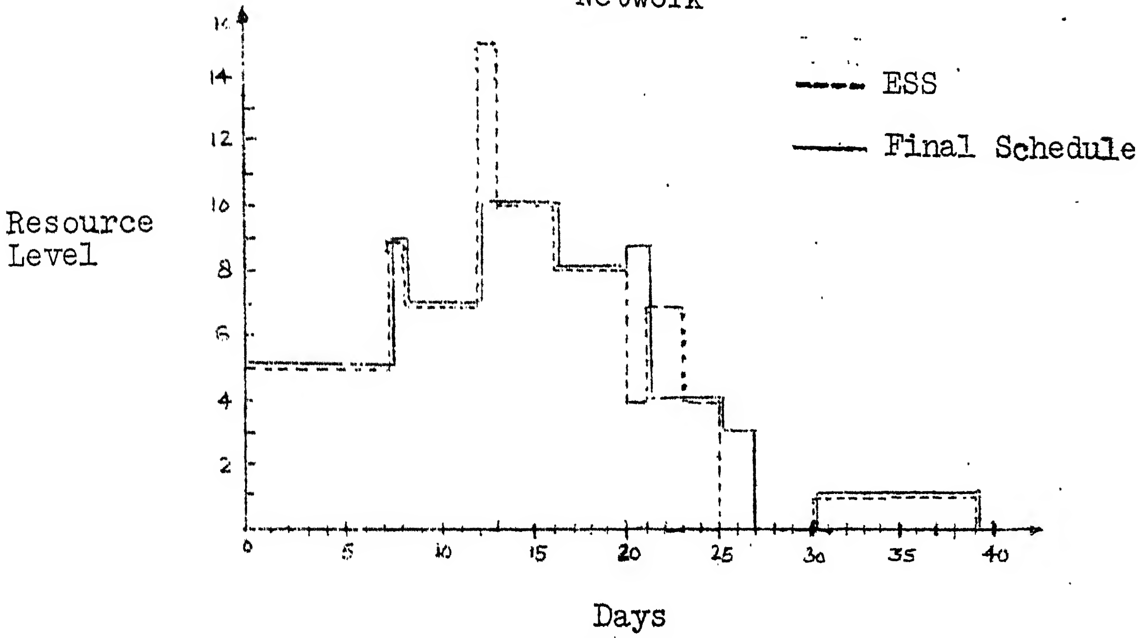


Resource 2

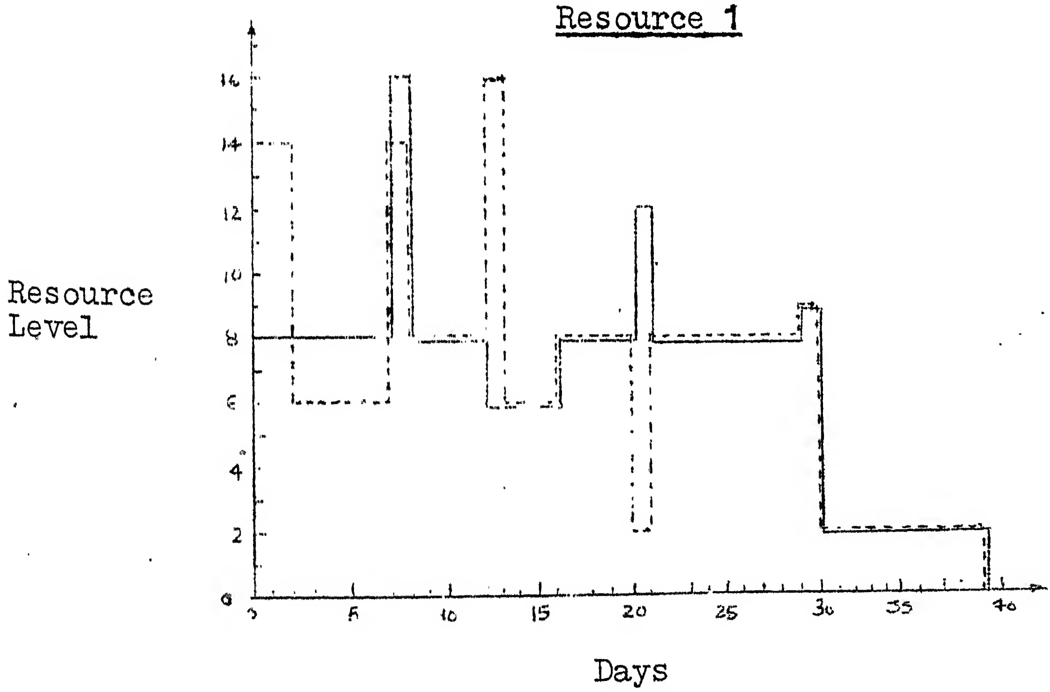
FIG. 1 : Resource demand patterns for Problem 8



Network

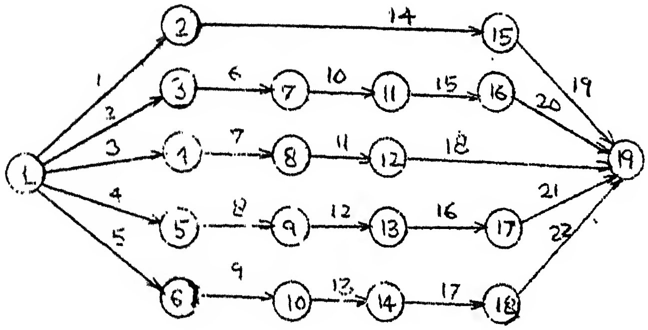


Resource 1

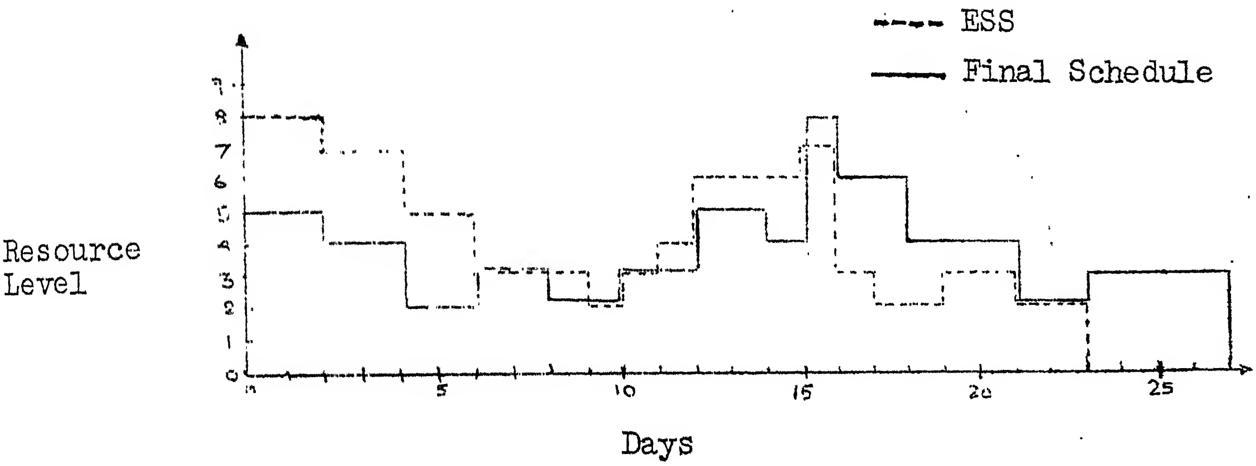


Resource 2

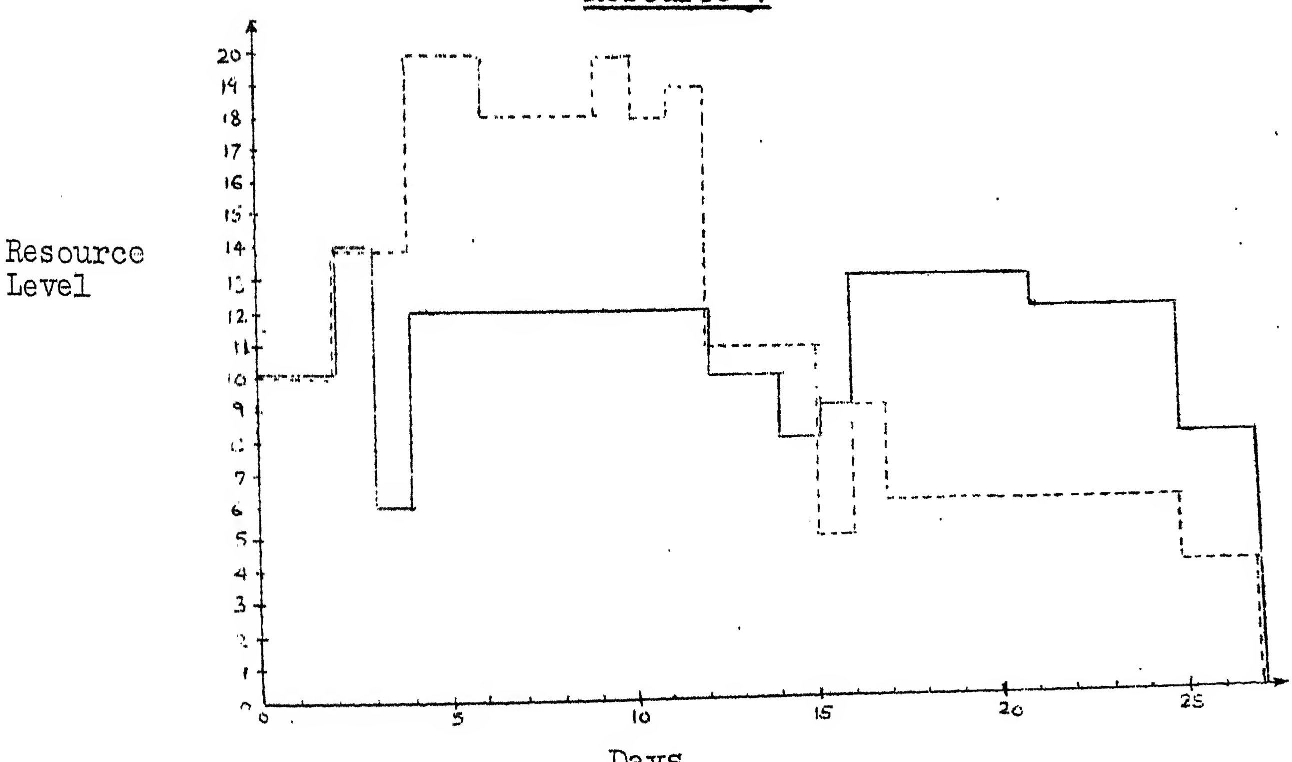
FIG. 2 : Resource demand patterns for Problem No. 7 and Policy 2.



Network



Resource 1



Resource 2

FIG. 3 : Resource demand patterns for Problem No. 9 and Policy 3.